New methodology to determine the terminal height of a fireball

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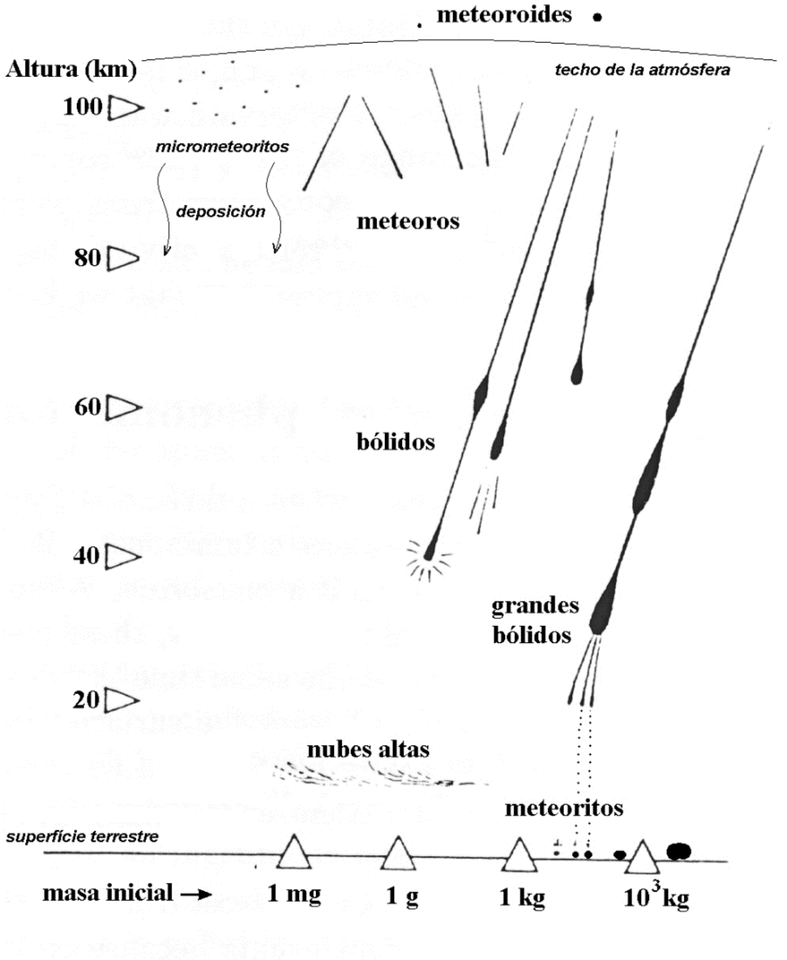
Summary

- Background
- Equations of motion
- Simplifications
- Database
- Results
- Discussion
- Conclusions
- References

- **Asteroid:** diameter > 10 m.
- **Meteoroid:** diameter < 10 m.
- **Meteor:** meteoroid that interacts with the Earth atmosphere.
- **Fireball:** meteor that is able to get deep in the atmosphere. Brightness similar to Venus.
- **Great Fireball:** bigger fireball that is able to get to lower altitudes and can reach a brightness similar to the Moon.
- **Meteorites:** meteor that survives to its atmospheric flight and reaches the ground.
- **Micrometeorites:** small grains that get the atmosphere at low velocities and are deposited on the ground .

Fireballs and Meteorites



Adapted from Rendtel et al. (1995)

Ceplecha y McCrosky (1976)

- Study of the Prairie Network to distinguish between ordinary and carbonaceous chondrites.
- The end height as the principal discrimating observational parameter in their discussion (*photometric mass*).
- Empirical Criterium:

$$PE = \log \rho_E + A \log m_\infty + B \log V_\infty + C \log(\cos Z_R)$$

- A, B and C obtained by Least–Squared fit on PN data.

Wetherill and Revelle (1981)

- Study of the Prairie Network to find similar meteorites to the Lost City meteorite.
- They used four criteria: End height agrees with the single-body theoretical value, calculated using *dynamic mass* , as well as with that of Lost City too within ±1.5km, when scaled for mass, velocity and entry angle in accordance with classic meteor theory.

Stulov et al. (1995), Stulov (1997) and Gritsevich (2007)

- Instead of using the average values as input parameters, they gathered all the unknowns into two new variables, α and β (ballistic coefficient and mass loss parameter).
- Adjusting the equation to the registered values these new variables can be obtained.
- This describes in detail the meteoroid trajectory and allow to invent a classification for possible impacts.

Equations of motion

1 The equations of motion for a meteoroid entering the atmosphere projected onto the tangent and to the normal to the trajectory

$$M\frac{dV}{dt} = -\overbrace{\frac{1}{2}c_d\rho_a V^2 S}^{\text{Drag force}} + P\sin\gamma$$

$$MV\frac{d\gamma}{dt} = P\cos\gamma - \frac{MV^2}{R}\cos\gamma - \overbrace{\frac{1}{2}c_L\rho_a V^2 S}^{\text{Lifting force}}$$

$$\frac{dh}{dt} = -V\sin\gamma$$

2 Variation of the mass

$$H^*\frac{dM}{dt} = -\frac{1}{2}c_h\rho_a V^3 S$$

3 Extra equations

$$Isotermal\ atmosphere \rightarrow \rho = \exp\left(h/h_0\right)$$

$$Levin\ (1956) \rightarrow S/S_e = (M/M_e)^{\mu}$$

4 Use of dimensionless parameters $M = M_e m; V = V_e v; S = S_e s; h = h_0 y; \rho_a = \rho_0\rho$

Where index e indicates values at the entry of the atmosphere.
h0 is the scale height (7.16 km)

$$\left.\begin{array}{l} Analytical\ \ solution\ of\ \ system: \\ Initial\ \ conditions: y=\infty; v=1; m=1 \end{array}\right\}$$

$$m=\exp\left(-\frac{\beta\left(1-v^2\right)}{1-\mu}\right) \quad [1]$$

$$y=\ln 2\alpha+\beta-\ln\Delta, \quad \Delta=\overline{Ei}(\beta)-\overline{Ei}(\beta v^2) \quad [2]$$

$$\overline{Ei}(x)=\int_{-\infty}^{x}\frac{e^z dz}{z}$$

In this methodology we gather all the unknown values of the meteoroid's atmosphere flight motion equations into two new variables (Gritsevich, 2009):

Ballistic Coefficient

$$\alpha=\frac{1}{2}c_d\,\frac{\rho_0 h_0 S_e}{M_e\sin\gamma}$$

Mass loss parameter

$$\beta=(1-\mu)\frac{c_h V_e^2}{2c_d H^*}$$

Simplifications

1

For quick meteors, a strong evaporation process takes place so β becomes high (β >> 1), the deceleration can be neglected and the velocity thus assumed constant. Stulov (1998, 2004) developed the following asymptotic solution:

$$v = 1, \quad m^{1-\mu} = 1 - 2\alpha\beta e^{-y}, \quad \ln 2\alpha\beta < y < \infty \quad [3]$$

2

However, the meteor velocity begins to decrease in a certain vicinity of m=0. In order to account for this change in velocity we combine the Eq.[1] (valid for arbitrary β values) with the Eq. [3] suitable for high β values:

$$v = \left(\frac{\ln\left(1 - 2\alpha\beta e^{-y}\right)}{\beta} + 1 \right)^{1/2}, \quad \ln 2\alpha\beta < y < \infty \quad [4]$$

Database

In order to test this methodology, we compare our derived terminal heights results with the fireball terminal heights registered by the Meteorite Observation and Recovery Project operated in Canada between 1970 and 1985 (MORP) (Halliday et al. 1996). We use previous α and β values derived by Gritsevich (2009).

1996M&PS...31..185H

TABLE 3. Physical and orbital data for a sample of 213 fireballs.

MORP	JD	Y	M	D	Δ	RA	DE	ZA	ZC	LH	LG	VI	VE	VH	VG	HB	HE	DU	a	e	i	q	q'	ω	Ω	S	DS	Q	MG	L	MI	MT	DN
132	2330.867	74	10	10	10	113.7	79.5	35.9	36.3	95.9	57.2	48.8	48.8	40.1	47.5	96.3	73.3	0.6	5.320	0.815	84.2	0.985	9.66	194.1	196.3		AI	1	9.9	2.88	29		
138	2356.825	74	11	05	0	17.9	-20.7	65.9	76.3	163.3	121.1	16.9	9.5	38.3	12.9	72.2	37.0	5.6	2.742	0.669	8.8	0.907	4.58	38.0	42.2		BK	2	8.7	3.83	2500	93	
141	2360.739	74	11	09	R	55.4	22.0	37.7	38.6	129.5	78.7	32.1	29.0	38.0	30.0	86.8	65.1	0.9	2.580	0.870	2.8	0.335	4.82	295.0	226.0	NT	AF	1	6.2	2.26	17		0.2
144	2362.801	74	11	11	20	26.7	40.1	16.9	18.5	147.4	98.0	23.5	13.5	39.3	21.4	74.9	42.9	1.6	3.550	0.836	16.5	0.720	6.38	247.3	228.2		E	1	7.1	2.83	140		0.8
167	2472.741	75	03	01	0	158.3	14.3	37.6	39.8	146.3	94.2	24.0	24.0	38.0	21.2	89.5	55.3	1.8	2.566	0.646	3.2	0.748	4.49	259.2	339.7	9	DG	1	10.7	4.33	3300		
169	2483.874	75	03	12	0	155.0	-14.6	69.8	75.0	148.2	96.7	22.9	9.3	38.0	20.2	78.9	34.0	6.4	2.620	0.726	13.7	0.729	4.50	68.7	170.8		CM	1	10.6	4.73	12000	62	
174	2542.892	75	05	10	24	47.6	71.4	47.2	52.3	155.7	103.7	19.1	12.3	36.4	15.4	72.1	32.3	3.3	2.050	0.555	19.9	0.913	3.19	137.0	48.8		BK	2	8.5	3.74	3900	2200	
177	2558.736	75	05	26	19	211.1	-6.8	50.6	59.9	166.0	121.8	15.8	8.7	36.6	11.2	69.6	39.8	3.3	2.151	0.570	1.7	0.926	3.38	220.2	64.1		G	3	6.7	2.85	300		3.2
178	2581.817	75	06	18	0	331.5	62.6	31.3	32.7	136.6	66.9	24.3	24.3	28.7	21.5	84.4	76.6	0.4	0.964	0.148	42.9	0.821	1.11	60.8	86.1		DI	1	9.9	3.04	160		
183	2629.815	75	08	05	R	36.9	56.4	41.8	42.0	68.6	40.9	60.0	60.0	41.3	58.8	100.1	80.1	0.4	22.449	0.958	112.0	0.951	43.95	150.7	131.9	P	A	1	7.0	2.30	5		
187	2654.818	75	08	30	6	320.9	6.1	46.2	52.2	155.7	100.7	18.4	14.0	35.4	14.8	73.2	46.3	2.3	1.756	0.557	9.1	0.779	2.73	248.9	156.0		AG	1	7.7	3.06	340		2.0
189	2669.831	75	09	14	9	319.2	-21.6	67.0	83.3	167.6	126.2	14.5	4.8	36.1	9.6	75.5	27.9	10.1	1.930	0.518	1.5	0.928	2.93	39.3	350.5		B	1	8.8	4.19	9100	970	
192	2677.669	75	09	22	3	253.4	52.5	30.0	33.1	152.5	102.8	21.0	14.6	38.0	18.0	71.3	44.4	1.6	2.710	0.632	27.2	0.998	4.43	170.2	178.2		A	1	7.6	2.66	120		
203	2724.052	75	11	07	0	56.7	22.0	61.8	64.3	127.1	75.6	32.3	25.9	37.2	30.6	88.9	67.2	1.5	2.168	0.867	2.8	0.288	4.05	301.9	224.1	NT	EH	1	9.2	2.89	75		
210	2734.047	75	11	17	R	152.1	23.7	29.7	29.9	20.0	11.5	70.2	70.2	40.2	69.3	104.2	84.4	0.3	5.013	0.803	160.0	0.987	9.04	175.7	234.2	L	A	1	8.7	2.85	13		
218	2757.733	75	12	11	29	60.1	15.7	32.6	36.2	157.9	107.3	18.5	13.2	37.4	14.7	70.7	37.9	2.3	2.185	0.635	2.0	0.798	3.57	59.1	78.3		A	1	6.2	2.64	140		4.2
225	2773.742	75	12	27	22	68.6	50.6	3.2	4.2	154.2	107.4	21.2	10.0	39.6	18.1	68.1	35.4	1.8	3.790	0.783	13.5	0.822	6.76	231.2	274.5		A	2	6.7	2.68	140		1.8
229	2805.702	76	01	28	22	359.4	34.4	51.6	66.0	172.0	142.3	12.3	6.1	34.7	5.6	63.8	40.7	3.3	1.484	0.344	4.4	0.973	2.00	162.5	307.0		A	2	5.4	2.42	150		2.2
232	2813.004	76	02	04	0	149.3	4.8	63.1	65.0	123.4	74.3	35.0	29.8	38.5	33.4	91.9	56.5	2.3	2.800	0.901	11.0	0.276	5.32	121.3	134.5		A	1	8.2	3.22	130		1.0
235	2830.627	76	02	22	31	131.1	83.1	27.7	30.5	158.4	112.5	19.1	17.5	38.8	15.5	82.6	60.7	1.3	3.050	0.680	20.6	0.975	5.12	195.2	332.3		G	2	9.6	3.44	720		
241	2877.788	76	04	09	30	176.2	16.6	33.6	39.0	162.6	117.6	16.7	9.8	37.3	12.6	67.0	34.9	2.6	2.320	0.610	4.8	0.903	3.73	222.4	19.2		A	1	7.5	3.25	800	71	
242	2899.794	76	05	01	17	210.5	65.3	15.0	16.5	156.5	111.0	19.9	19.9	38.7	16.5	87.4	67.8	1.0	3.365	0.702	23.1	1.004	5.73	188.2	40.7		EG	1	7.9	2.71	120		
248	2989.823	76	07	30	R	29.6	57.2	35.2	35.3	72.1	43.4	59.4	59.4	42.0	58.2	96.5	79.9	0.4	-	1.020	108.4	0.955	-	151.9	126.9	P	EF	2	10.5	3.45	71		
251	2995.824	76	08	05	R	37.9	54.3	40.7	40.9	65.0	38.0	60.7	60.7	40.9	59.5	97.3	82.1	0.3	11.430	0.916	115.5	0.954	21.90	151.2	132.6	P	A	1	8.4	2.56	9		
253	3002.770	76	08	12	R	44.3	58.4	49.4	49.7	69.0	41.0	60.0	60.0	41.3	58.8	88.2	75.8	0.3	19.351	0.950	111.9	0.965	37.74	154.5	139.3	P	EI	2	8.6	2.56	9		
265	3102.022	76	11	19	0	149.2	17.2	32.4	32.5	8.7	5.0	71.0	71.0	40.1	70.1	102.2	81.5	0.3	4.750	0.793	172.2	0.985	8.52	187.2	237.0	L	E	1	10.4	3.34	38		
272	3126.952	76	12	14	0	115.3	31.0	25.6	26.5	114.1	61.2	36.2	32.4	33.2	34.6	88.9	49.7	1.2	1.269	0.904	23.1	0.122	2.42	327.7	262.2	G	EG	1	10.0	3.60	290		
278	3142.051	76	12	29	7	97.4	63.7	39.3	42.5	147.9	92.6	22.4	22.4	36.8	19.6	67.8	53.3	0.8	1.969	0.612	22.1	0.764	3.17	245.6	277.6		B	2	9.5	3.62	750		
284	3173.972	77	01	30	20	106.1	-0.8	80.4	90.4	157.8	110.3	18.8	18.8	38.5	15.5	93.2	80.5	4.1	2.750	0.692	9.8	0.848	4.65	48.7	130.1		A	1	4.7	2.27	48		
287	3181.807	77	02	07	11	135.7	13.5	36.7	39.2	146.8	93.3	23.4	10.5	37.5	20.6	69.6	34.6	2.2	2.250	0.717	2.1	0.637	3.86	81.2	138.1		AP	1	7.6	3.21	350		3.5
291	3199.631	77	02	25	24	137.1	18.9	40.8	45.1	159.1	111.3	18.5	18.5	37.9	14.5	91.1	62.5	2.0	2.480	0.661	0.9	0.841	4.12	231.7	336.0		FL	1	9.3	3.61	1100		
300	3241.832	77	04	08	24	190.2	0.6	47.5	51.7	150.5	97.9	21.5	10.2	37.1	18.5	70.4	46.2	1.9	2.235	0.680	2.6	0.715	3.76	252.9	18.0		AR	1	7.5	2.82	160		0.4
301	3242.875	77	04	09	17	217.3	-13.9	63.1	64.9	118.2	68.6	36.1	24.4	36.3	34.4	95.3	55.6	2.6	1.960	0.905	1.4	0.186	3.74	315.4	19.1		A	1	6.7	2.72	44		0.6
304	3251.894	77	04	18	2	184.6	16.5	47.7	56.2	163.3	118.1	16.2	6.6	36.9	12.0	69.0	36.8	3.5	2.180	0.582	5.6	0.914	3.45	221.0	27.9		E	1	8.0	3.36	960		2.5
309	3286.869	77	05	23	24	310.2	39.7	29.6	29.8	92.4	55.0	49.9	49.9	39.8	48.5	97.2	72.3	0.6	5.209	0.806	87.6	1.012	9.41	181.4	61.7		AF	3	6.9	2.29	7		
310	3299.809	77	06	05	15	228.2	47.9	12.8	15.7	160.7	112.5	17.0	10.5	36.3	13.0	76.1	36.8	2.6	2.070	0.516	18.4	1.002	3.14	195.7	74.1		BK	1	7.6	2.99	390		2.2
314	3341.857	77	07	17	34	240.9	-1.3	66.7	82.4	170.5	141.1	14.6	9.3	37.5	9.8	70.4	42.1	5.6	2.599	0.620	5.0	0.989	4.21	201.7	114.3		F	2	6.0	2.89	480	65	
317	3366.824	77	08	11	R	45.6	57.5	41.8	42.0	66.5	40.1	61.8	61.8	42.6	60.6	100.6	84.3	0.3	-	1.068	114.1	0.953	-	152.3	138.1	P	A	1	6.1	1.72	1		
325	3397.734	77	09	11	18	46.2	36.4	57.9	58.2	51.6	30.6	64.3	64.3	40.5	63.1	103.3	82.7	0.6	7.117	0.914	141.9	0.612	13.62	259.7	168.0		A	1	8.6	2.80	14		
331	3441.882	77	10	25	6	166.6	18.5	61.8	82.0	169.9	39.6	13.3	7.0	25.0	6.9	71.4	31.1	7.1	0.757	0.338	3.2	0.501	1.01	18.5	211.7		EL	2	9.1	4.13	10000	2500	
333	3450.749	77	11	03	14	100.8	49.4	44.2	44.5	71.4	41.4	58.2	58.2	39.8	56.9	114.0	90.1	0.6	4.275	0.890	117.4	0.472	8.08	276.5	220.3		A	1	7.7	2.53	9		
336	3455.787	77	11	08	0	151.2	29.8	79.5	79.8	31.1	18.0	69.8	69.8	41.2	68.7	110.5	83.2	2.1	9.328	0.898	150.4	0.952	17.70	156.7	225.4		G	1	9.1	3.60	73		
339	3458.756	77	11	11	24	112.1	32.6	57.6	57.9	53.6	32.0	66.0	66.0	42.4	64.8	114.1	87.5	0.8	-	1.002	154.7	0.432	-	277.2	228.4		A	1	9.0	3.14	28		
340	3458.759	77	11	11	R	57.1	14.2	40.6	42.0	134.5	81.1	29.1	25.6	37.1	26.8	88.1	69.1	0.9	2.126	0.810	5.9	0.405	3.85	108.7	48.4	ST	EG	1	7.9	2.48	35		
346	3469.709	77	11	22	17	16.7	48.7	4.3	6.3	162.5	108.2	15.7	8.5	35.2	11.1	64.1	31.0	2.3	1.600	0.443	11.5	0.892	2.31	226.8	239.4		E	3	8.6	3.50	1500	160	
349	3482.980	77	12	05	R	122.3	4.0	45.3	45.7	72.6	42.2	58.0	58.0	40.1	57.0	100.4	66.8	0.8	4.585	0.952	128.4	0.218	8.95	126.6	72.8	7	DH	2	10.0	3.51	86		
379	3725.807	78	08	05	R	36.9	58.2	42.9	43.2	70.8	42.6	60.0	60.0	42.2	58.8	97.9	81.8	0.4	-	1.033	109.8	0.952	-	151.6	132.1	P	A	2	10.4	2.98	24		
380	3728.811	78	08	08	R	37.9	57.4	39.1	39.3	69.8	41.0	58.6	58.6	40.2	57.4	98.1	83.4	0.3	6.489	0.851	110.6	0.966	12.01	153.8	135.0	P	A	1	7.2	2.17	4		
382	3730.852	78	08	10	R	43.5	56.2	32.5	32.6	66.3	39.2	60.2	60.2	40.8	59.1	93.0	75.9	0.3	10.088	0.906	114.3	0.952	19.22	150.7	137.0	P	EF	3	9.5	2.74	14		
383	3731.896	78	08	11	0	39.6	58.0	25.8	25.8	68.7	41.0	60.0	60.0	41.4	58.9	89.8	81.7	0.2	25.800	0.962	111.5	0.981	50.50	159.1	138.0	P	EF	1	9.0	2.28	5		
384	3734.896	78	08	14	23	317.9	-23.5	76.1	83.3	150.8	98.2	21.1	12.1	36.6	18.1	80.3	53.8	5.8	2.166	0.677	4.0	0.699	3.63	76.5	320.8		A	1	6.8	2.99	280		0.8

Table 1b.

The results of processing of observational data for the Canadian Network fireballs.

Fireball No.	Ve [a], km/s	α	β	$10^2σ$	sin γ	M_1, g	M_2, g	M_{ph} [a], g
18	18.5	24.13	1.475	0.862	0.572	5417.24	11466.73	5600
123	16.3	37.22	1.111	0.836	0.551	1651.41	3495.55	680
138	16.9	38.90	2.889	2.023	0.404	3669.82	7767.96	2400
141	32.1	707.19	6.403	1.243	0.767	0.09	0.19	17
144	23.5	137.13	1.301	0.471	0.953	6.38	13.51	140
169	22.9	50.52	1.575	0.601	0.337	2886.41	6109.70	12,000
172	12.5	13.13	3.379	4.325	0.969	6916.28	14639.77	1200
177	15.8	77.63	1.777	1.424	0.631	121.19	256.52	300
187	18.4	83.76	3.815	2.254	0.663	83.17	176.06	340
189	14.5	34.47	0.757	0.720	0.388	5954.14	12603.20	8100
192	21.0	94.01	2.458	1.115	0.860	26.95	57.05	120
195	25.2	35.22	1.486	0.468	0.593	1563.53	3309.54	7000
204	13.0	10.13	3.620	4.284	0.537	88488.54	187304.65	2900
205	19.7	36.84	0.716	0.369	1.000	284.89	603.03	6300
207	17.9	24.45	0.775	0.484	0.927	1223.38	2589.55	3400
218	18.5	64.65	0.339	0.198	0.820	95.61	202.37	140
219	18.4	12.51	2.060	1.217	0.772	15813.00	33471.56	87,000
223	27.1	18.33	1.809	0.493	0.326	66757.19	141305.66	232,000
225	21.2	75.92	0.706	0.314	1.000	32.55	68.90	140
229	12.3	43.73	4.564	6.033	0.623	704.43	1491.07	150
231	27.9	52.72	1.371	0.352	0.384	1716.79	3633.96	4300
232	35.0	434.10	0.000	0.000	0.447	1.95	4.13	130
235	19.1	171.01	14.343	7.863	0.887	4.08	8.64	720
241	16.7	50.27	0.878	0.630	0.826	198.96	421.15	730

Halliday et al., 1996 *Gritsevich, 2009*

Resolution

Adjusting the (vi, yi) values of Eq. [2] to the trajectory observed (vi, yi) values by means of a weighted least-squares method. Assigning manually the weighted factors may be quite complicated, so, since the height and velocity of a meteor decrease while it gets closer to the surface, the solution was proved to perform better if we take an exponential form of eq. [2] (see Gritsevich, 2008 for further details):

$$2\alpha \exp(-y) - \Delta \exp(-\beta) = 0, \quad \Delta = \overline{Ei}(\beta) - \overline{Ei}(\beta v^2)$$

$$\overline{Ei}(x) = \int_{-\infty}^{x} \frac{e^z dz}{z}$$

We can derive these new variables (α, β) for each meteoroid by minimizing this expression.

$$Q(\alpha, \beta) = \sum_{i=1}^{n} (F_i(y_i, v_i, \alpha, \beta))^2$$

$$F_i(y_i, v_i, \alpha, \beta) = 2\alpha \exp(-y_i) - \Delta_i \exp(-\beta) = 0$$

From Eq.[3], at the point were m=0 we have:

$$h_I = h_0 y_t = h_0 \ln 2\alpha\beta \quad [5]$$

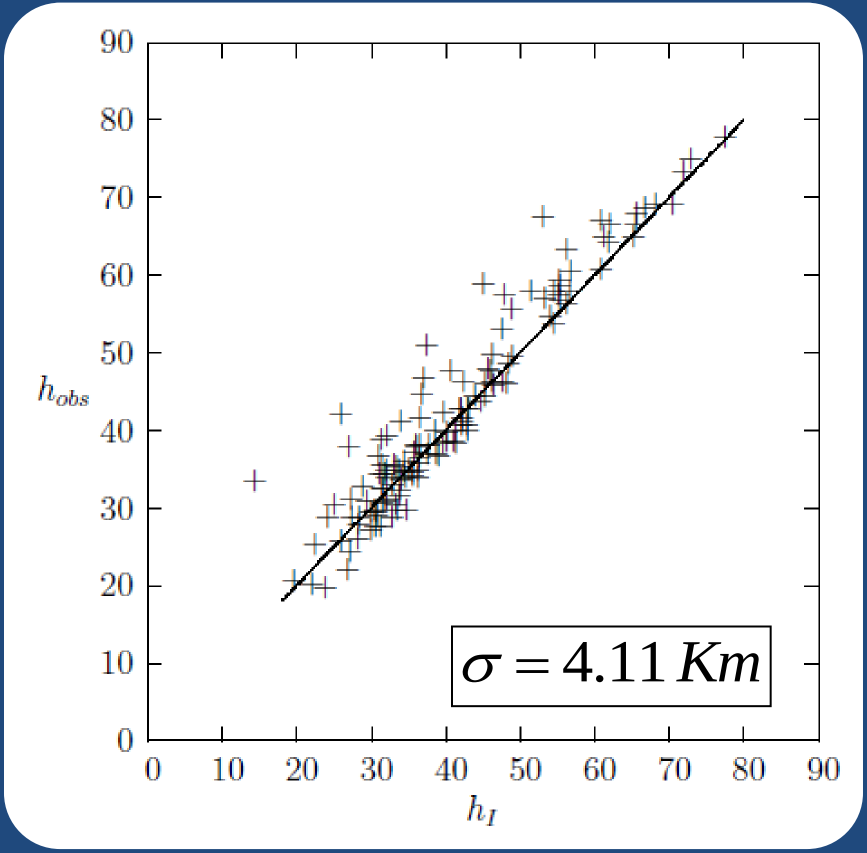
If we reorder Eq.[4] we have:

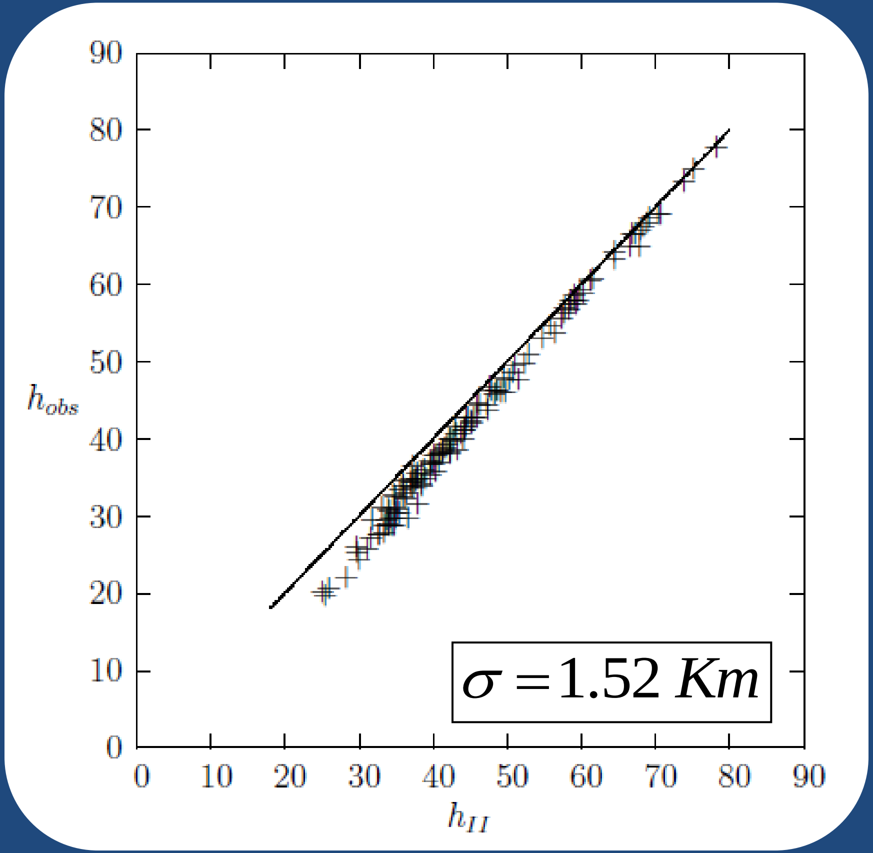
$$h_{II} = h_0 y_t = h_0 \ln\left(\frac{2\alpha\beta}{1 - e^{\beta\left(v_t^2 - 1\right)}}\right) \quad [6]$$

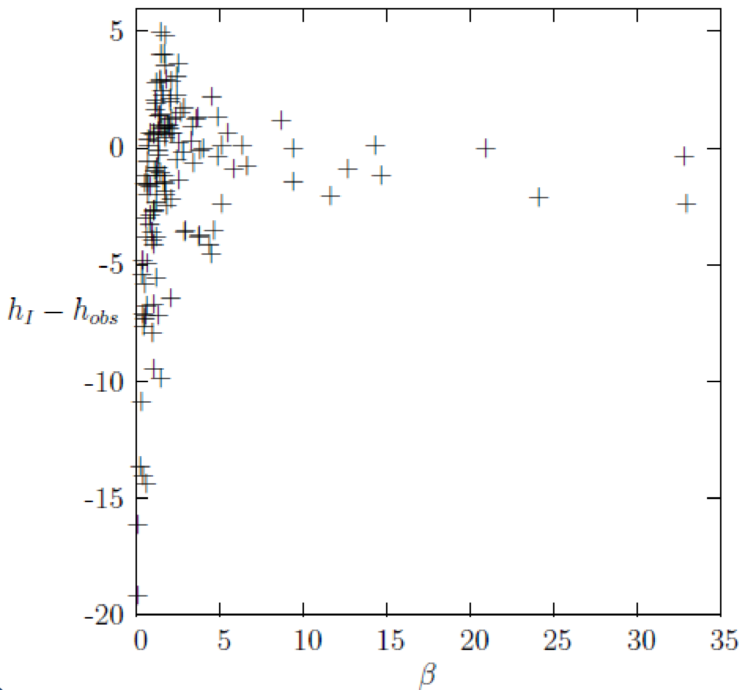
Results

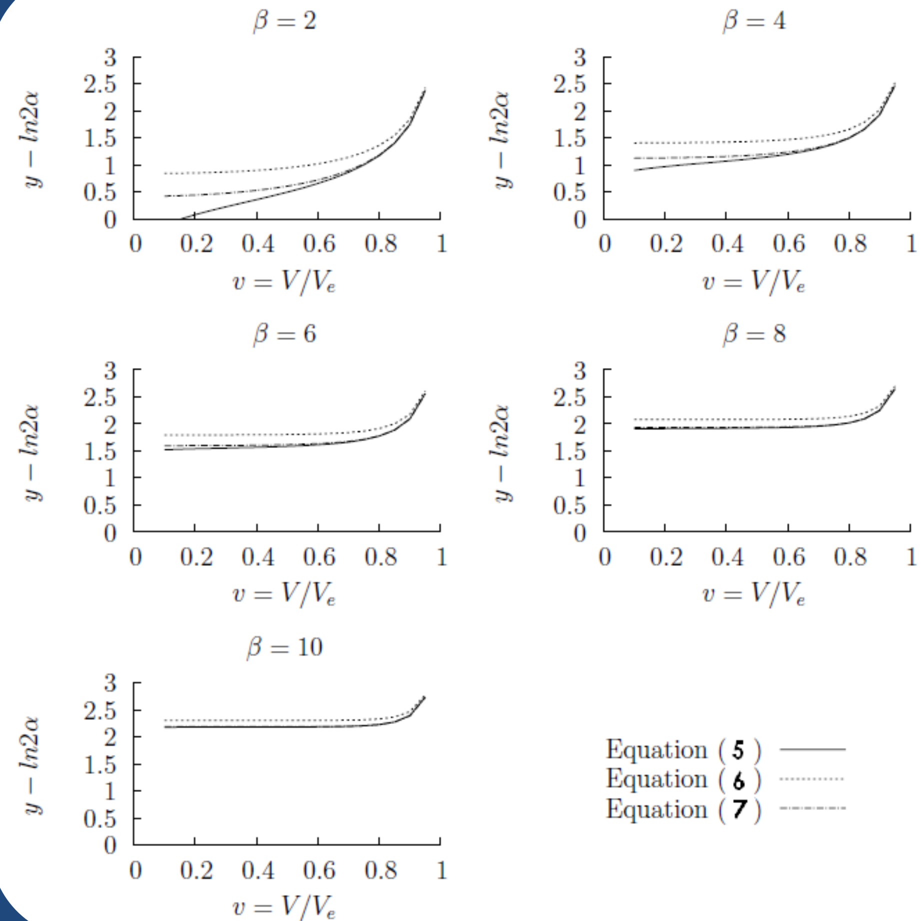
$$h_I = h_0 y_t = h_0 \ln 2\alpha\beta$$

$$h_{II} = h_0 y_t = h_0 \ln\left(\frac{2\alpha\beta}{1-e^{\beta\left(v_t^2-1\right)}}\right)$$







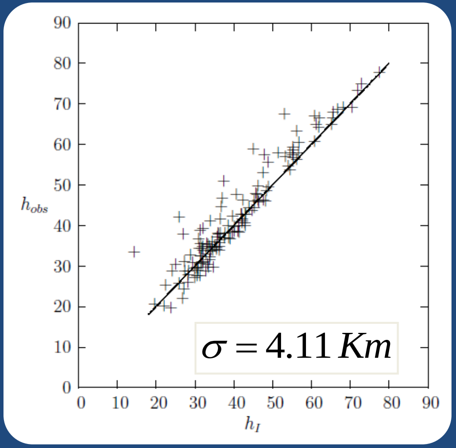


Differences could be due to the use of eq. [4] established for high β.

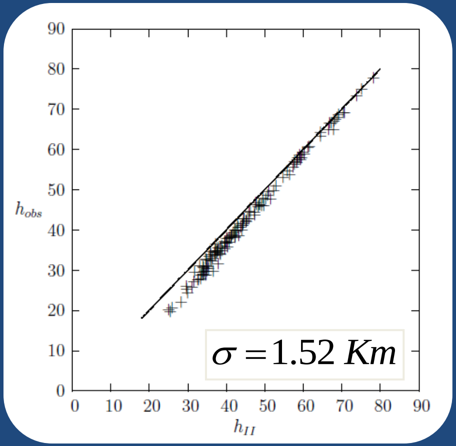
As suggested by Gritsevich et al. (2015), we would rather use the modification: $\beta \rightarrow \beta - 1.1$

$$h_{III} = h_0 y_t = h_0 \ln\left(\frac{2\alpha(\beta - 1.1)}{1 - e^{(\beta-1.1)(v_t^2 - 1)}}\right) \quad [7]$$

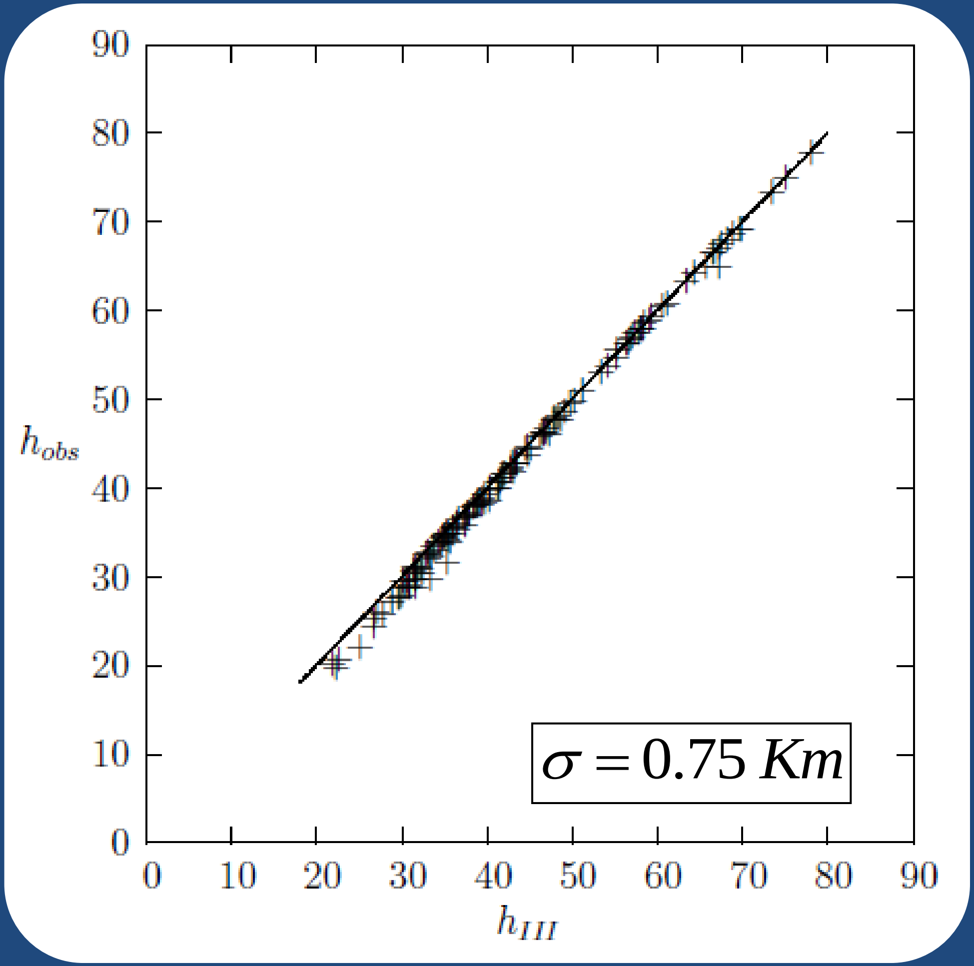
$$h_I = h_0 y_t = h_0 \ln 2\alpha\beta$$



$$h_{II} = h_0 y_t = h_0 \ln\left(\frac{2\alpha\beta}{1-e^{\beta\left(v_t^2-1\right)}}\right)$$



$$h_{III} = h_0 y_t = h_0 \ln\left(\frac{2\alpha\left(\beta-1.1\right)}{1-e^{\left(\beta-1.1\right)\left(v_t^2-1\right)}}\right)$$



Discussion

For $\beta > 5$ eq. [5] shall give good results. But the decrease in v near the terminal point is not considered.

Eq.[6] shows a lineal tendency. Still discrepancies in all β values.

The modification made in [7] leads to a good agreement between observed and theoretical data. However at low β, some discrepancies appear.

The small discrepancies at low β shall be taken into account for future planetary defense applications. Meteoroids could reach lower height than those predicted.

The inverse problem is possible for non decelerated bodies if the terminal height is known → constraints in α and β.

Meteor height as a function of time → new problems may be scoped:
- Determination of luminous efficiency based on meteor duration.
- Critical Kinetic Energy to produce luminosity.

We use dimensionless parameters instead of the empirical set A,B, C coefficients of Ceplecha and McCrosky (1976). However α and β keep the same variable dependency as the PE criterium.

- We have derived the terminal heights for MORP fireballs using a new developed methodology.
- This methodology had only been tested on several fully ablated fireballs with large β values (Gritsevich and Popelnskaya, 2008).
- We were particularly interested in determining whether this new mathematical approach works equally for fully ablated fireballs and meteorite-producing ones.
- We introduced a new modification in the methodology which allows to get a higher accuracy.
- We foresee a calculation of terminal height to be useful when the lower part of the trajectory was not instrumentally registered.
- It also brings critical knowledge into the problem when one needs to predict how long will be a total duration of the luminous flight or at which height a fireball produced by a meteoroid with given properties would terminate.
- Based on our investigations we can highly recommend the use of equation [7] also to solve inverse problem when terminal height and velocity are available from the observations, and parameters α and β need to be derived.

References

M. Moreno-Ibáñez, et al., Icarus 250, 544-552 (2015).

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Z. Ceplecha and R.E. McCrosky, J.Geophys. Res. 81, 6257-6275 (1976).
M. Gritsevich, Solar Syst. Res. 41 (6), 509-514 (2007).
M. Gritsevich, Solar Syst. Res. 42, 372-390 (2008).
M. Gritsevich and N.V. Popelenskaya, Doklady Phys. 53, 88-92 (2008).
M. Gritsevich, Adv. Space Res. 44. 323-334 (2009).
M. Gritsevich et al., Mat. Model. 27 (2) 25-33 (2015).
I. Halliday et al., Meteorit. Planet. Sci. 31, 185-217 (1996).
V.P. Stulov et al., Aerodinamika bolidov, Nauka (1995).
V.P. Stulov, Appl. Mech. Rev. 50, 671-688 (1997).
V.P. Stulov, Planet. Space Sci. 46, 253-260 (1998).
V.P. Stulov, Planet. Space Sci. 52 (56), 459-463 (2004).
G.W. Wetherill and D.O. Revelle, Icarus 48, 308-328 (1981).